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Exploratory Hypersonic Boundary-Layer Transition Studies

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Nomenclature

h = distance normal to surface
M = Mach number
Pc = inviscid cone static pressure
P_t = pitot pressure
$P_{t,\infty}$ = freestream stagnation pressure
R = Reynolds number
S_t = Stanton number
T = temperature
u = velocity
x = axial distance from cone tip
δ^* = displacement thickness
θ = momentum thickness

Subscripts

e = edge of boundary layer
w = wall
∞ = freestream

An exploratory investigation to determine the feasibility of making boundary-layer transition studies at freestream Mach numbers on the order of 20 in the Langley 22-in. helium tunnel has indicated that 1) transition Reynolds numbers in the 30–40 $\times$ 10^6 range exist with local Mach numbers of about 15, 2) the transition Reynolds number at hypersonic speeds, at least for very slender axisymmetric configurations, is extremely sensitive to angle of attack, and 3) the thin-film resistance gage, which customarily is used to measure heat-transfer rates in impulse facilities such as the shock tunnel, is a useful device for measuring heat-transfer rates in relatively long running-time tunnels when heat-transfer rates are very low.

The model used in the exploratory investigations was a sharp-nosed (nose diameter = 0.003 in.) 5-ft-long cone with a base diameter of 6 in. (cone half-angle of 2.87°) and was constructed of Fiberglas. M_∞ varied with $P_{t,\infty}$ from a low of about 19 at 500 psi to a high of about 23 at 4000 psi. Model wall temperature was always about 540°R. Stagnation temperature was about 540°R during pitot-pressure measurements and about 820°R during heat-transfer measurements.

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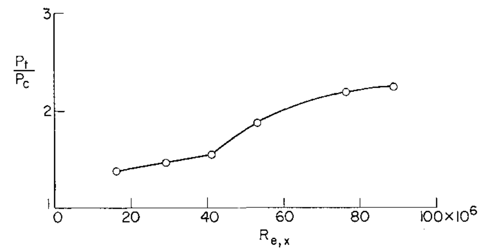

Fig. 1 Variation of surface pitot to wall static pressure ratio with Reynolds number at x = 58 in., $T_w/T_t \approx 1$.

The model initially was instrumented with surface thermocouples, as this was intended to be a temperature recovery model. However, the heat-transfer rates at M_∞ = 20 were so low that, for the run times available (about 40 sec), no useful data were obtained.

A surface pitot probe, 0.060 in. in diameter, was installed at x = 58 in., and data were taken at stagnation pressures ranging from 500 to 3600 psi. These pitot pressures have been ratioed to the inviscid cone static pressure and are shown in Fig. 1 plotted against $R_{e,x}$. Transition appears to initiate at a Reynolds number of approximately 40 $\times$ 10^6.

Additional information concerning the state of the boundary layer then was determined by a boundary-layer pitot-pressure survey at the same station. These data were reduced to velocity ratio, assuming constant total temperature and static pressure through the boundary layer, and they are presented in Fig. 2 along with the resultant δ^* and $R_{e,\theta}$.

The highest Reynolds number profile appears to be turbulent, though it is not certain that this represents a fully developed turbulent flow. The inner half of the lowest Reynolds number profile appears laminar; the outer half has the earmarks of turbulence. The variation in shape of the velocity profile with Reynolds number suggests that turbulence is initiated in the outer region of the boundary layer and moves in toward the wall as Reynolds number increases; that is, turbulence may exist in the boundary layer long before its effect is felt at the wall. This is in line with the observation of others with respect to the variation of the location of the critical layer with increasing Mach number (see, e.g., the discussion in Ref. 1, p. 42).

Prior to obtaining the data of Fig. 2, an attempt was made to obtain a pitot survey using eight tubes simultaneously during a run. The tubes were spaced 45° around the periphery of the model. There was a large amount of scatter in these data because of nonsymmetry of the boundary layer, although the model angle of attack was within ±0.2° of zero, as determined by 4 static pressure orifices in the model 90° apart. The nonsymmetry is attributed to the effects

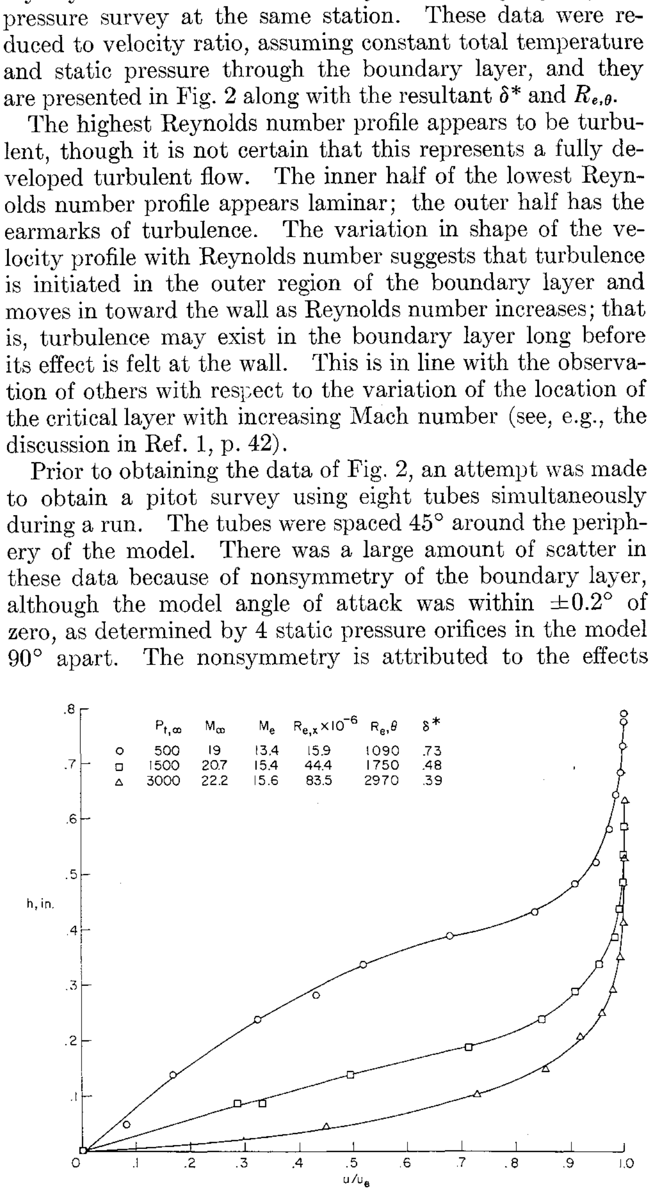

Fig. 2 Velocity ratio distribution at x = 58 in., $T_w/T_t \approx 1$.

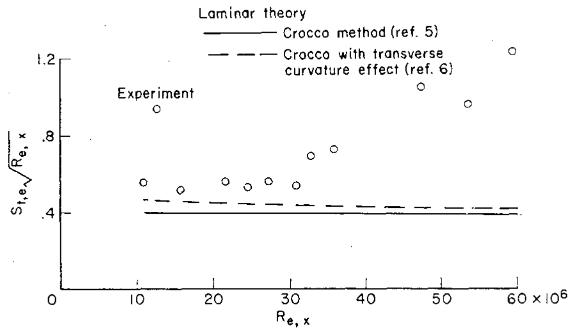

Fig. 3 Variation of heat-transfer parameter with Reynolds number at $x = 58$ in., $T_w/T_t \approx 0.66$.

of slight flow angularity inducing three-dimensional effects, which enhance flow instability and the onset of turbulent bursts.

Thin-film resistance gages then were installed in the same model, and heat-transfer tests were performed over a stagnation pressure range of 500 to 4000 psi. Results of this test for the heat gage at $x = 58$ in. is shown in Fig. 3. Here, a Reynolds number for transition of roughly 30×10^6 is indicated. It should be mentioned that between each of the tests described herein the model was removed from the tunnel and reinstalled. Thus, the question of alignment is pertinent. In addition, $T_w/T_t \simeq 0.66$ for the heat-transfer tests, whereas $T_w/T_t \simeq 1$ during the pitot-pressure measurements. Although all of the data are for angles of attack within $\pm 0.2°$ of zero, the range of Reynolds number for transition indicated here $[30(10)^6$ to $40(10)^6]$ is felt to be primarily a result of the extreme sensitivity of transition Reynolds number for this configuration to angle of attack rather than a result of wall temperature effects. Sensitivity of transition Reynolds number to angle of attack also has been noted previously at lower Mach numbers (see, e.g., Refs. 2–5). Although a transition Reynolds number range of $30(10)^6$ to $40(10)^6$ appears high, it should be noted from Fig. 2 that, at $R_{e,x} = 44(10)^6$, $R_{e,\theta} = 1750$ (which compares with $R_{e,\theta} = 1000$ for transition at $M = 10$ in Ref. 2).

The curves shown in Fig. 3 are theoretical estimates. The solid line is the Crocco method solution[6] for the laminar boundary layer on a flat plate modified by the Mangler transformation for flow over a cone. The dashed line is the transverse curvature correction to the Crocco solution predicted by Ref. 7 and is seen to be significant. Even with the transverse curvature correction, the theory is 15 to 20% below the experimental results, however. The extent to which the difference is a result of experimental error, entropy gradient effects from shock curvature caused by boundary-layer displacement effects near the cone tip, the three-dimensional nature of the boundary layer, or inadequacy of boundary-layer theory in the hypersonic range, is not known at present. It is hoped that future work will shed some light on this aspect of the problem.

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Numerical Treatment of Turbulent Flows

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It is in the nature of numerical models of hydrodynamic flow problems that a model should substitute finite approximations for the functions describing the hydrodynamic and thermodynamic state of the flow. A usual technique is to divide the region of flow into a finite mesh of grid points at which functional values are specified. The differential equations of hydrodynamics then are approximated by difference equations involving these grid-point values. Presently available computers permit calculations with 10^4 grid points to be carried out for 10^3 time steps in tens of hours.

It is clear that, in any finite numerical model of a turbulent hydrodynamic flow, we must be satisfied with an explicit description of only those scales of motion larger than the grid scale. All of the scales of motion smaller than grid scale must be treated, if at all, as more or less random motions superimposed on the large scale flow. Thus the appropriate division between "mean" and "eddy" flow is determined by grid size. Even though the initial state of a fluid involves only larger scales of motion, the nonlinear nature of the equations leads to motions of smaller and smaller scale. For a numerical model to be realistic it too must lead to a cascading of energy from larger to smaller scales reaching, usually quite soon, the scale of the grid. It then is necessary to rely on some statistical treatment of the further cascade process. This process can be considered as loss of energy from the larger scales of motion and thus as an energy dissipation. Perhaps the simplest solution in lieu of a greater understanding of turbulence is to introduce a viscosity to provide this dissipation. It is tempting also to consider this viscosity as an eddy viscosity describing the statistical influence of the subgrid scale motions on the explicitly described flow. Such an eddy viscosity was introduced by Smagorinsky[1] in a numerical model of the atmosphere.

The magnitude of the eddy (kinematic) viscosity coefficient ν should depend on the grid scale λ and on the specific energy cascade rate ϵ. If it depends only on these quantities, then dimensional considerations determine its form. The viscosity coefficient ν has dimensions of L^2T^{-1}, a specific energy cascade rate ϵ of L^2T^{-3}, and a scale λ of L. Letting $\nu = \alpha\epsilon^m\lambda^n$ with α dimensionless, we have $L^2T^{-1} = L^{2m}T^{-3m}L^n$, whence $m = \frac{1}{3}$, $n = \frac{4}{3}$, and $\nu = \alpha\epsilon^{1/3}\lambda^{4/3}$. We have assumed that the molecular viscosity is negligible and plays no role in determining the eddy viscosity. Should the molecular

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